Volume xx (200y), Number z, pp. 1–19

Semi-regular Triangle Remeshing: a Comprehensive Study

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Abstract

Semi-regular triangle remeshing algorithms convert irregular surface meshes into semi-regular ones. Especially in the field of computer graphics, semi-regularity is an interesting property because it makes meshes highly suitable for multiresolution analysis. In this paper, we survey the numerous remeshing algorithms that have been developed over the past two decades. We propose different classifications to give new and comprehensible insights into both existing methods and issues. We describe how considerable obstacles have already been overcome, and discuss promising perspectives.

Categories and Subject Descriptors (according to ACM CCS): I.3.5 [Computer Graphics]: Computational Geometry and Object Modeling—Curve, surface, solid, and object representations

1. Introduction

Advances in acquisition and computer graphics have helped the development of numerical representations of surfaces in many domains: computer-aided design and mechanical engineering, digital entertainment, augmented and virtual reality, medical imagery, simulations, architecture and so on. One of the most popular representation is the polygon mesh. Despite significant advances in the processing of quadrangle meshes, triangle meshes are still considered as a standard representation: they are simple, flexible and widely supported by graphics hardware.

The size of treated meshes grows continuously. It is motivated by the quality requirements of end applications such as rendering. It is supported by increasing capacities for storing and processing massive data, which are provided by high resolution acquisition devices, and computer-aided design and modeling techniques. Huge meshes require complex data management (e.g. streaming) even when using workstations. It becomes more constraining by the expansion of connected and/or hand-held devices with limited memory and bandwidth. Distributed storage and processing also stimulate the demand for compact representations. Moreover, the variety of devices, usages and contexts stresses the need for scalable representations. In this context, semi-regular meshes are a well-suited representation because of their good scalability and compactness properties. The focus on semi-regular (SR) meshes has been first motivated by the efficiency of wavelets in signal processing [Mal89]. Compared to irregular meshes, they offer a piecewise regular structure, and are thus better suited for multiresolution analysis. Their ability to enable modeling, analyzing and rendering surfaces at different levels of resolution is essentially based on the efforts in the late 1990s and early 2000s to adapt wavelets to surfaces [SS95, DDSW95, CPD*96, FW96, LDW97, KS00, KSS00, Ber04, LQS04, CS08, KPA12].

The actual use of SR meshes depends on their availability. They may be produced either by meshing of other types of data (*e.g.* point clouds or implicit surfaces), by direct semi-regular modeling, or, most of the time, by remeshing of irregular meshes. It has been an active field of research for two decades and new challenges have been addressed in recent years. Research is driven by a few main objectives a remesher must meet in order to make the output SR meshes suitable for later applications. The geometric and visual fidelity to the irregular input mesh is one of them. A second one is the quality of the output, which relates the shape of the triangles and the sampling of the vertices. A third one is the compactness of the output in terms of memory. These goals are inspired by many applications related to signal processing and computer graphics.

Most applications take advantage of the ability of SR meshes to represent data at different resolutions. It is hence possible to efficiently navigate between the resolutions for level-of-detail visualization and rendering [CPD*96] or to interactively edit, animate or apply special effects [ZSS97, BMBZ02, SHB07]. In this case, fidelity to the input is desirable at all resolution levels. Some other applications exploit the properties of the wavelet analysis in both the spatial and frequency domains: segmentation [RDB07], watermarking [WLDB08], or view-dependent processing (coding, visualization, transmission) [GAB04,SKKL04,PMA09, Rou10]. The compactness is a goal for SR remeshing because of applications in geometry compression [KSS00, LMH00, KG03, LCB03, AFX04, PA05, SKKL04, DSM*10] and progressive transmission [CPD*96, LKSS00]. Since the construction of semi-regular meshes relies on a parameterization of the original mesh on a coarser one, some applications, like mesh morphing [LDSS99, MKFC01], texture or detail transfer [PSS01, BMBZ02] make use of this parameterization. Most of these applications are part of a processing pipeline that ends with a rendering step. Thus controlling the shape and size of the triangles is important. In this pipeline the remeshing is considered as a pre-processing step which does alter the shape, so fidelity is a central goal of all remeshing algorithms.

We propose in this paper a comprehensive study of the semi-regular triangle remeshing techniques which have been developed over the last two decades. It is intended for readers with various areas of interest (functionalities, technical details, usability). Therefore, we explain and classify the methods from different standpoints. We highlight the overcome problems as well as the emerging challenges.

To take further advantage of this survey, we suggest several complementary readings about closely related topics that we do not detail here: surface remeshing [AUGA08], mesh parameterization [SPR06, HPS08], subdivision surfaces [Cas12], quad mesh generation/processing [BLP*13], and compression [AG05, PKK05].

In Section 2, we first remind the readers of the basics about semi-regular meshes and multiresolution analysis. We also set up the very context of the present study by bringing out a common structure for all semi-regular remeshing algorithms. Then, we review the chronology, highlighting three major trends. Based on these general considerations, we compare the algorithms from different perspectives in the following sections.

In Section 3, we propose a classification of the methods according to the goals. In addition to general remeshing goals (shape fidelity and mesh quality) we investigate the compactness as a specific goal for semi-regular remeshing. Each algorithm is looking for a trade-off between these possibly conflicting goals.

Section 4 is structured around the technical components of the algorithms. It compares the different ways of performing the main tasks, and thereby stresses connections between methods.

Section 5 focuses on input and output data properties.There the reader can consider remeshing as a black-box which must meet specific requirements.

Finally, we summarize the most important issues discussed across the survey, raise some others, and outline emerging challenges (sections 6 and 7).

2. Background

2.1. Semi-regular meshes and multiresolution analysis

The regularity of a mesh refers to the connectivity of its elements. A triangle mesh is said to be semi-regular (SR) if it follows a specific structure: the triangles can be merged by fours down to a low resolution (LR) mesh (see red triangles in figure 1). As a result, all the vertices are regular (*i.e.* have valence 6) except the vertices of the LR mesh. A convenient way to build a SR mesh consists in refining a mesh by 1-to-4 splits, as for primal subdivision schemes. Therefore, SR meshes are said to have “subdivision connectivity”.

This structure is suitable for a wavelet-based multiresolution (MR) analysis of the mesh geometry (see figure 1). A multiresolution analysis defines different levels of resolution of the SR mesh. The high-resolution mesh can be recursively analyzed by:

- a low-pass filter to get a mesh at a lower level of resolution;
- a high-pass filter to get a set of details which encodes the geometrical difference between two consecutive resolutions.

In this way, the mesh can be expressed as a low resolution mesh (representing low frequencies) and a series of details (each one representing a band of frequencies). Any detail or vertex can thus be localized both in frequency (its resolution) and space (its position) leading to a space-frequency analysis.

The reverse of the *analysis* process is called *synthesis* (figure 1, from right to left). At each level:

- a subdivision scheme is applied to get a mesh at a higher resolution;
- details are added to get high frequencies.

Both analysis and synthesis are implemented with local stencils, just as subdivision is. A major progress in implementation has been the lifting scheme [Swe98]: it allows for efficient in-place computations and easy tuning of the filters.

A wavelet basis [Mal99] underlies this MR analysis scheme: the details are wavelet coefficients, while the positions of the vertices are scaling coefficients. Wavelet bases

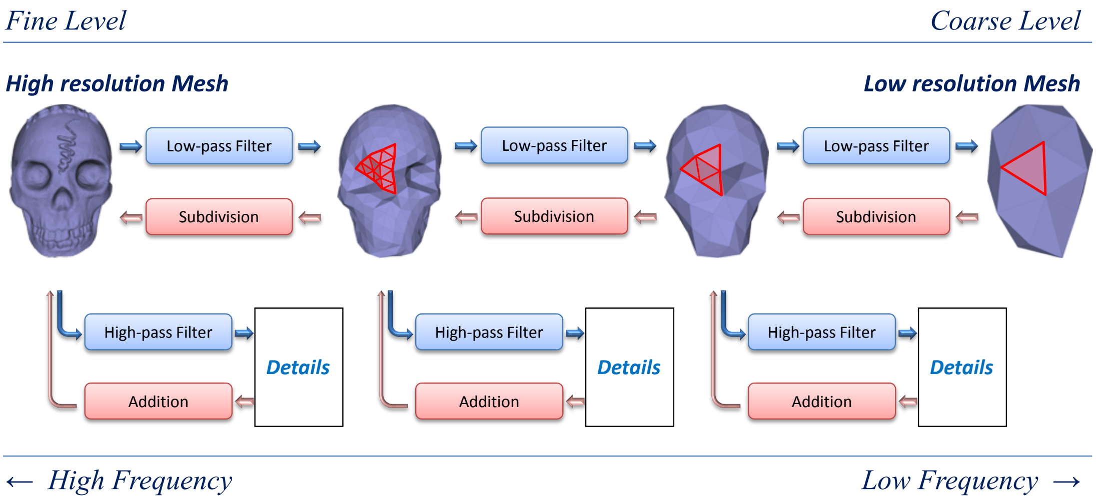

Figure 1: *Multiresolution analysis of a semi-regular mesh. From left to right: a high resolution mesh is recursively analyzed, which produces a low resolution mesh and a series of details located in both the spatial and the frequency domains. From right to left: the synthesis process subdivides the mesh (see red triangles) and adds details at each level.*

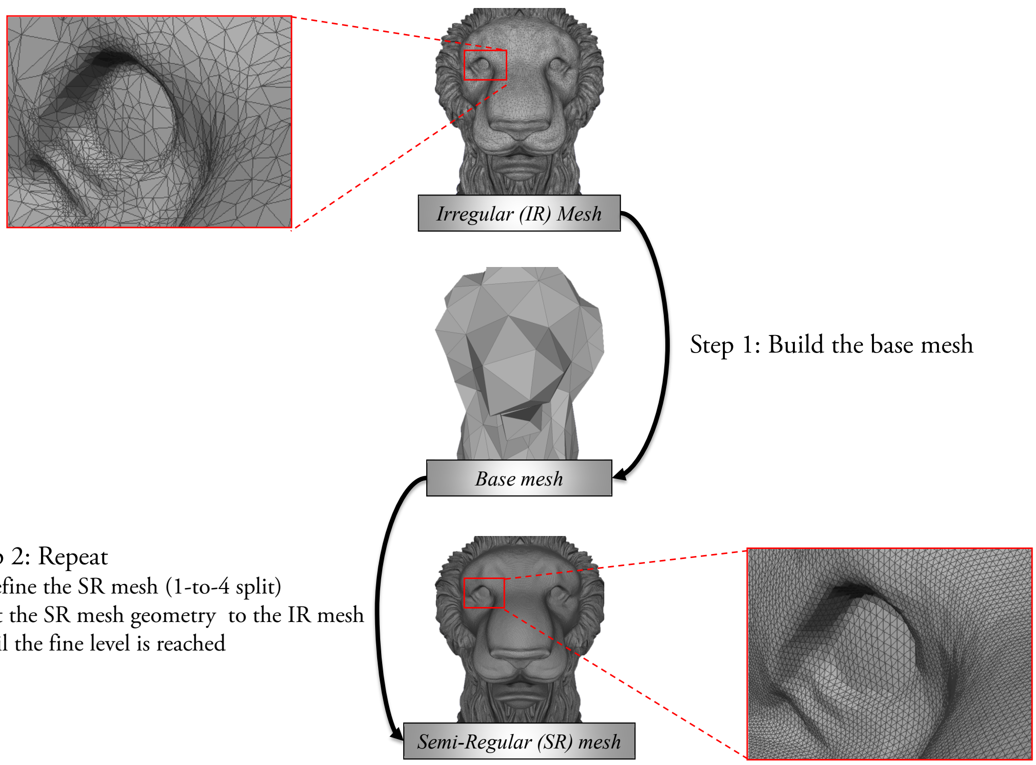

Figure 2: *Overview of semi-regular (SR) remeshing.*

have been first defined on regular grids (*e.g.* images). Although several wavelet transforms have been developed for irregular meshes [KCVS98, Bon98, GSS99, VP04, RFK*05], the semi-regular meshes remain particularly suitable for multiresolution analysis.

It is useful to notice that, from the wavelet theory point of view, the SR mesh is a *control* mesh through which a continuous surface can be manipulated. This surface can be defined in two ways: as a linear combination of wavelets and scaling functions; as the limit of the subdivision process applied to the high resolution (it is consequently called *limit surface*). This surface thus depends on the MR scheme, and it is smooth in the vast majority of cases. On the contrary, in most computer graphics applications, the SR mesh is itself considered as the final surface. This distinction is enlightening on some upcoming issues.

Semi-regular meshes have two main interesting properties. First, their connectivity is entirely defined by the low resolution (LR) mesh. Indeed, the connectivity of finer levels is implicit and does not need to be encoded. Second their multi-scale structure enables MR analysis, and so the control on localization and resolution simultaneously.

2.2. Semi-regular remeshing overview

A semi-regular (SR) remesher takes an irregular (IR) mesh as input and computes a SR mesh that approximates the IR mesh. All the algorithms follow the structure described by figure 2. First, a base mesh is built: topologically, it is a 2-manifold simplicial complex which is used as parametric domain in most methods; geometrically, it is a coarse approximation of the IR mesh. The SR mesh is then computed iteratively from the base mesh by alternating refinement and geometric fitting onto the IR mesh. The fitting stage typically uses a parameterization of the IR mesh on the base mesh, in order to compute a geometric position for any vertex inserted by refinement, with respect to the IR mesh.

Choices for these components greatly impact on the objectives that the remesher can and do achieve. Both description of the components (section 4) and discussion of the goals (section 3) often resort to the concept of *patches* illustrated in figure 3. A patch is the part of the IR mesh defined (implicitly or explicitly) as the image of a base triangle through the parameterization. Significant efforts are devoted to the design, the study and the optimization of the patches.

One notices that the remeshing process (step 2 figure 2) and the MR synthesis (from right to left figure 1) are alike: to reach the fine level, both of them repeatedly split the triangles and replace the vertices. However they must be carefully distinguished: the synthesis applies reversible linear filters on the SR mesh, while remeshing is a pre-processing which involves complex algorithms and a reference IR mesh. As a consequence, the base mesh (middle figure 2) and the LR mesh (right figure 1) must also be distinguished: they have

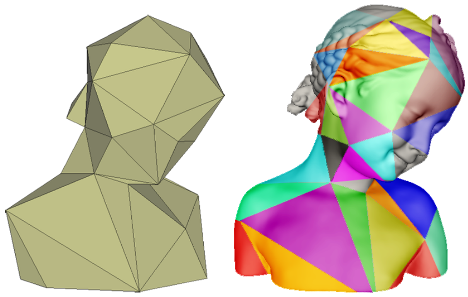

Figure 3: *A base mesh (left) of* BIMBA *and the associated patches (right). Each colored region corresponds to one patch,* i.e., *the part of the IR mesh assigned to a base triangle through the parameterization.*

the same connectivity but they differ in their geometries, depending on both the remeshing algorithm and the MR analysis filters. This is also valid for intermediate levels of resolution. Though they are similar, confusing the remeshing and the analyzed mesh hierarchies may hinder deep understanding of the remeshing issues.

We attract the reader's attention to the connection with parameterization. Almost any global mesh parameterization method can be used for SR remeshing, provided that the parameter domain can be coarsely triangulated (*e.g.* [AMD02, SAPH04]). Geometry images can also be used [GGH02, PH03, SWG*03] since they mainly act as a global parameterization. However, considering only the parameterization is not sufficient. It is rather a tool among others in the remeshing process, which contributes to the achievement of some objectives. Therefore, we do not investigate parameterization methods in details. The interested reader can refer to [FH05, SPR06].

2.3. History

The interest for SR remeshing emerged in the mid-1990s with Lounsbery *et al.*'s work about multiresolution analysis for surface meshes [Lou94, LDW97]. Starting from a simple polyhedron as base mesh and a linear subdivision scheme, the authors defined wavelets on SR meshes. Transforming an IR mesh into a SR one became necessary to fully exploit the promising wavelet transform on surfaces.

During a first period running from 1995 to 2000, works about SR remeshing mainly focused on the design of base meshes and local correspondences between the base mesh and the IR mesh. The challenges were the handling of arbitrary genus, boundaries and complex shapes [EDD*95, LSS*98, Gio99], and the compactness of the representation [GVSS00, LMH00]. The fitting of SR vertices onto the IR mesh was processed through local parameterization or local projection procedures.

At that time, the resulting SR meshes exhibited artifacts, especially along the patch boundaries. Lack of parameterization smoothness was considered as the major cause of these artifacts. Therefore, from 2001 to 2010, most researchers worked towards reaching smooth and low-distortion parameterizations [HLG01, KLS03, FSK04, Gus07]

Since 2010 a third period has been marked by challenging sampling problems, such as aliasing and feature preserving. The new goal is geometric fidelity rather than mesh quality. It has been noticed that smooth parameterization is not enough: it may even hinder the sampling of high frequencies. The emerging trend is based on a direct resampling of the IR mesh in $\Re^3$ [DMS10, KPA10, CJL11].

3. Goals

Among the main issues arising during any remeshing process [AUGA08], only two are generally discussed for SR remeshing and considered as goals: shape fidelity and quality of the mesh. Since SR meshes are accepted to be an efficient model for compression, we also propose to investigate the compactness.

In the following we examine these 3 goals: what they mean, how they are evaluated, how they are achieved, and how they conflict. The discussion is kept general concerning the underlying techniques, which are defined and detailed in section 4. The second column of table 1 (page 6) summarizes the contributions to these three goals of all the SR remeshing methods. One should notice that only contributions (not the absolute performances) are rated here. In this way, we emphasize where the efforts have been focused rather than showing that earlier methods are out-of-date.

3.1. Shape fidelity

The SR mesh has to be a good approximation of the shape defined by the IR mesh. The precision is primarily measured by a geometric distance between the IR mesh and the SR mesh at its fine and/or intermediate levels. Preserving geometric features is also relevant.

3.1.1. Remeshing error

All the methods tend to minimize a global *remeshing* error. It is mostly evaluated by the mean square error (L_2-norm) between the SR mesh at its finest level and the IR mesh. The maximal error (L_∞-norm) is rarely considered [EDD*95, Gio99, Gus07]. In the following we discuss how remeshing algorithms achieve low error: what tools are used, how the error is controlled, and whether or not intermediate levels are taken into account.

The basic strategy to reach low error is to build a roughly uniform sampling of the IR mesh. To further minimize the error, three tools are mainly considered in the literature: adaptiveness, anisotropic remeshing, and sampling optimization.

The first strategy to lower the remeshing error is adaptive remeshing [LSS*98, GVSS00, HLG01, KPA10]. The principle is to refine the triangles only where it significantly decreases the remeshing error. It tends to reduce the number of triangles in flat or smooth regions (low frequencies) compared to detailed regions (high frequencies). Thus it capitalizes on the space-frequency localization which is provided by the MR structure.

Anisotropy has been proposed only by Guskov [Gus07]. Indeed, it is well-known from approximation theory that anisotropic remeshing whose elements are aligned and sized with respect to the curvature tensor field asymptotically optimally approximates a shape [ACSD*03, CSAD04].

The third strategy consists in optimizing the position of the SR vertices (see discussion in section 4.3) by approximating the IR mesh [FSK04], by optimizing the sampling in high frequency regions [DMS10] while interpolating the IR mesh, or by using a Voronoi tessellation directly in 3D space during the refinement [KPA10].

Then, there is a standard, simple, and greedy approach to control the error: the base mesh is refined until the computed remeshing error is below a given tolerance. Lee *et al.* [LSS*98] additionally guarantee a logarithmic bound on the number of levels, but cannot in any case predict the precise number. An alternative would consist in minimizing the error for a given triangle budget, which is however interesting only for adaptive remeshing. Theoretical guarantees on the construction of an optimal piecewise linear approximation for a given number of elements are still lacking, especially concerning coarse simplifications [CSAD04].

In some applications it is of importance that not only the fine level but intermediate levels as well are faithful. For instance, intermediate levels are used as geometric approximations in progressive transmission or level-of-detail rendering. Thus some methods [FSK04, Gus07, KPA10] tend to reduce the error at every level.

Remember however that meshes at intermediate resolutions may differ between remeshing and the later MR analysis (see section 2.2). To make them identical, two assumptions are needed. First during remeshing, the vertex positions must not change after the geometric fitting stage: changes happen for instance during relaxation procedures used for parameterization or for sampling optimization. Second the vertices must not move during the later MR analysis, which requires the MR scheme to be interpolating. Of course it can not be guaranteed because it is independent of the remeshing. Note however that the most popular MR schemes are actually interpolating because they derive from butterfly subdivision [DLG90].

A particular point must be raised for the method proposed in [FSK04], which is the only one that approximates the IR mesh during the construction of the SR mesh. On one hand, their remeshing strategy is particularly relevant since the ge-

	Goals (contributions)			Components (methods)			Input and output (features)				Remarks
Ref.	Shape fidelity	Mesh quality	Compact-ness	Base mesh	Parameterization	Geom. fitting	Any Genus	Bnd.	Adapt.	Sharp Features	
[EDD*95]	+	+		MP	local harmonic	FI	✓	✓			
[LSS*98]	++	+		IS	conformal	FI	✓	✓	✓	✓	a.k.a. MAPS
[Gio99]	+		+	PP	local harmonic	FI	✓	✓		✓	based on [EDD*95]
[KVLS99]		+		PP	implicit	FI					
[GVSS00]			++	IS	shape-preserving	FI	✓		✓		a.k.a. *normal meshes* or INM
[LMH00]			++	IS	no param.	FI			✓		a.k.a. *displaced subd. surfaces*
[HLG01]	+	++		PP	MIPS	FI		✓	✓		
[KLS03]		++	+	IS	conformal	FI	✓	✓			based on [LSS*98], a.k.a. GSP
[LKK03]				IS	shape-preserving	FI	✓	✓			extension of [GVSS00] for boundaries
[FSK04]	++		+	N/A	N/A	A	✓	✓			param. as input, based on [GVSS00]
[AGL06]				IS	conformal	FI	✓	✓		✓	OoC extension of [LSS*98]
[LYHL06]		+		PP	min. area disto.	FI	✓	✓			
[Gus07]	++	+	+	MP	mean-value	FI	✓	✓			a.k.a. TriReme, anisotropy
[PTC10]		++	+	IS	conformal/authalic mix	FI	✓				
[KPA10]	+	+		MP	conformal	VI			✓		
[DMS10]	+		+	N/A	N/A	VI					SR meshes as input
[CJL11]	+	++		MP	N/A	FI	✓			✓	

Table 1: *Comparison of semi-regular triangle remeshing methods.*
Goals: *rate the contribution (∅/+/++) to each remeshing goal. Notice that it does not rate the absolute performance.*
Components: *building of the base mesh (IS = incremental simplification, MP = mesh partitioning, PP = parameter domain partitioning), parameterization type, geometric fitting (FI = face interpolation, VI = vertex interpolation, A = approximation).*
Input and output: *check if the features are provided (arbitrary genus, mesh with boundaries, adaptive output, preservation of sharp features).*
Remarks: *OoC = Out of Core.*

ometrical distance between the IR and the SR meshes is minimized at each level of resolution during refinement. On the other hand, during MR analysis, no scheme can guarantee that the intermediate resolutions will be identical to the ones produced during remeshing.

3.1.2. Feature preservation

Preserving surface features is important for the visual quality, because they contribute to the overall aspect of the shape. It is particularly true for sharp features (sharp edges or corners) where aliasing can occur, if the sampling is not aligned with those features or not dense enough (see figure 4). The ability to preserve features is supported in the literature by convincing examples, but standard geometric errors fail to measure it. Figure 4 illustrates this: local errors are averaged downwards by the L_2-norm because of the integral over the surface; errors with low magnitude (L_∞-norm) may ruin sharpness as well as smoothness. Recent advances in perceptual metrics [CLL*13] could be helpful here.

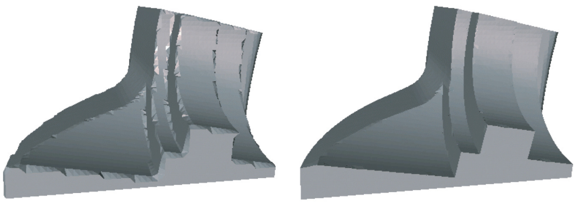

Figure 4: *Aliasing on sharp edges greatly damages the appearance though low errors are measured with standard geometric distances (L_2 and L_∞). Images of [AUGA08].*

Lee *et al.* are the first to propose a preservation of user defined features [LSS*98]. The user can tag some IR vertices or edges, to make their removal impossible during the creation of the base mesh. Later, two methods automatically preserve sharp features [Gio99, CJL11], by segmenting the original surface in regions that are as flat as possible. The region boundaries match the sharp features and are then preserved during the simplification process, leading to the creation of the base mesh. It eases their preservation at the intermediate levels during remeshing. Thus during analysis also, intermediate meshes will exhibit the sharp features provided that an interpolating scheme is used (as discussed in section 3.1.1).

Notably, the problem of sharp features preservation is ill-posed. There is even no simple definition of these features because they are located on edges and at vertices, where the field of mesh normals is not continuous. Thus, unlike for smooth surfaces, discontinuities in the derivatives can not characterize the features. So their characterization relies either on prior knowledge, or on empirical rules. An alternative could lean on the wavelet theory, relying on the (smooth) limit surface rather than on the mesh (see section 2.1). This would however trip on the opposite problem: apart from the basic linear scheme, subdivision is designed to avoid discontinuities everywhere.

3.2. Quality of the mesh

When focusing on the quality of a mesh, one is interested in both local (per triangle) and global (sampling) properties:

- A **triangle** is called **well-shaped** if it is close to equilateral.
- A **sampling** is **uniform** if the density of vertices is constant over the surface. It results in triangles with homogeneous sizes.
- A **sampling** is said to have **smooth gradation** if the density changes are progressive along the surface.
- A **sampling** is **isotropic** if the density does not depend on any direction.

In the SR remeshing setting, one is interested in how the finest level of the hierarchy meet these criteria. It mainly depends on how the base mesh is built, and then how the SR vertices are fitted with the initial surface during refinement. These technical issues are discussed in section 4. Here we focus on the assessment and on global issues.

SR meshes would be ideally composed of well-shaped triangles with uniform sampling, but this is challenging [KCS98]. Hence the sought quality depends mostly on the intended application. For compression purposes, uniform sampling is sought [AUGA08, SPR06] because it leads to wavelet coefficients with small tangential components which can be efficiently compressed. In numerical simulation, the angles of every triangle (regardless of their size) should not be too large, since it can affect both accuracy and efficiency [She02]. In this case, well-shaped triangles and smooth gradation are favored. For rendering the sampling density matters, especially on silhouettes. Elongated triangles are also detrimental to interpolate correctly the normals, which is important for lighting.

3.2.1. Shape of the elements

Mainly four measures are used to discriminate well-shaped from degenerated triangles. As stated in [HP11], the *aspect-ratio* has been derived from the general definition for convex bodies: ratio between its longest and shortest dimensions. It is generally defined for triangles as the length of the longest side divided by the length of the shortest height (*i.e.* distance from the longest edge to its opposite vertex). It can also be mistaken for the *edge ratio* (the ratio between the longest and shortest sides of a triangle, that equals to one for equilateral triangles), or the *radius-edge ratio* (the ratio of the circumradius to the shortest edge length). Finally, dealing with the evaluation of surface meshes for finite element analysis [FB97], some people also estimate the ratio of the inner circle radius over the length of the longest edge (normalized so as to equal one in case of an equilateral triangle). This measure is called *roundness* in [KCS98].

Until [Gus07, CJL11], the quality of the SR mesh elements is only assessed visually. Roundness and triangle area distributions are studied in [Gus07] to discuss some input parameters. Lately, Chiang *et al.* [CJL11] use aspect-ratio distributions to compare their results with previous works. Moreover, some authors [KLS03, CJL11] make use of the roundness to enhance the quality of the resulting SR meshes, thanks to relaxation procedures. But no theoretical guarantee on the shape of elements has been provided for SR remeshing while it has been studied in the irregular setting [CDS13].

3.2.2. SR sampling

Obtaining uniform sampling is challenging. During refinement, subdivision ensures a *parametrically* uniform vertex distribution within each patch. This is however not sufficient to generate a globally *geometric* uniform sampling since i) the distortion introduced when parameterizing onto the base mesh can affect intra-patch re-sampling and ii) the variation of the patch sizes may imply strong variation in the sampling density across the patch boundaries.

Obtaining isotropy and smooth gradation is more affordable. The main issue is to avoid artifacts along patch boundaries (see figure 5). Most of the time, this problem is overcome by optimizing a parameterization: smoothing in the parametric domain [LSS*98, KVLS99, HLG01]; local updates [GVSS00,PTC10]; or global updates [KLS03,Gus07]. Another approach consists in using a Voronoi tessellation of the initial surface to position the SR vertices [KPA10].

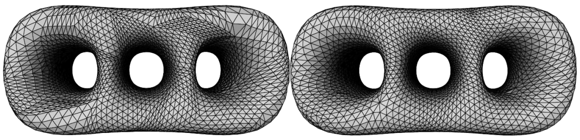

Figure 5: *Left: A parametrically uniform sampling within each patch does not necessarily involve a globally geometric uniform sampling. Right: a smooth gradation sampling reduces artifacts along patch boundaries. Image of [LSS*98].*

Moreover, some complex shapes contain small "topological features" such as handles or holes. Preserving the topology requires small base triangles around these features. In such a case non-uniform sampling makes it possible to avoid a too fine base mesh while smooth gradation prevents boundary artifacts from appearing.

3.3. Compactness

Compactness is the ability of a surface representation to encode large objects with few data. Actual encoding involves a compact connectivity representation and an efficient compression technique applied on the geometry information. Semi-regular meshes are efficient in terms of compactness because of two reasons: First, the connectivity information is limited to the triangles of the base mesh, which involves a negligible quantity of connectivity information. Thus standard connectivity coding algorithms [TG98, Ros99] are often considered as sufficient though it is still an active field of research [GLLR13]. Second, SR meshes support wavelet analysis which significantly improves the compression of the geometry information. Khodakovsky *et al.* [KSS00] showed that encoding SR meshes with wavelet-based algorithms decreases the reconstruction error by a factor four compared to other progressive coding schemes (even if the triangle count is generally higher, compared to IR meshes).

Some SR remeshing techniques explicitly focus on compactness, to further improve the coding performances. It consists in optimizing the trade-off between quality of the encoded data and size of the resulting binary file. This is assessed by comparing the bitrate-PSNR curves of the SR meshes encoded with a progressive wavelet coder, *e.g.* [KSS00]. The bitrate relates to the file size, often given in bytes per IR vertex. The PSNR measures (in dB) the quality of the SR mesh, progressively encoded and decoded. It is given by $PSNR = 20log_{10}\frac{BB}{RMS}$, with BB the length of the bounding box diagonal, and RMS the root mean square error computed between the IR and the SR meshes. This error can be estimated with some available software tools, *e.g.* MESH [ASCE02] or METRO [CRS98].

The underlying idea for improving the coding performances is to create SR meshes such that the sets of wavelet coefficients obtained by MR analysis are as sparse as possible. To this end, [Gio99] improves on [EDD*95] by a partition-based construction of the base mesh, which is specially designed to minimize the energy of the subsequent wavelet coefficients. Base triangles represent as-flat-as-possible patches and sharp features are maintained in the base mesh, such that the reconstruction error for a given budget of wavelet coefficients is reduced.

Another popular way to improve the coding performances consists in building a smooth parameterization with low distortion [KLS03]. Indeed, such parameterizations are "regular" functions which are known from the wavelet theory to lead to wavelet coefficients with a rapid decay (w.r.t. frequency) and thus to a sparse representation. A corollary (specific to surfaces) is that it results in uniform sampling, so the tangent components of the wavelet coefficients are small, which also helps in compression.

Other methods instead tend to optimize the SR sampling. The objective is to generate wavelet coefficients that can be represented by one single scalar value, instead of the usual 3D vector. The so-called *normal meshes* [GVSS00, LKK03, FSK04] and *displaced subdivision surfaces* [LMH00] result from such an approach. The key contribution of [GVSS00] is the fitting method: each vertex (inserted by subdivision) is moved along its normal direction until "piercing" the original surface. Provided that the same subdivision scheme is used during remeshing and MR analysis (actually the modified Butterfly [ZSS96]), most details are scalar-valued. This implies that *normal meshes* enable significant coding gains in comparison with *non normal* SR meshes [KG03]. However, using another subdivision scheme during MR analysis (*e.g.* Loop [Loo87]) lowers compression ratios because tangential components appear.

At the same time, Lee *et al.* [LMH00] aim to describe *all* the SR vertices with scalar-valued displacement maps from a smooth surface, which makes the building of the base mesh crucial. This method may be more efficient than [GVSS00], but computing such a base mesh is challenging in particular with non smooth models.

However, forcing as many vertices as possible to move along the normal directions (during the geometric fitting) tends to increase the remeshing error. Friedel *et al.* relax this constraint if it results in a SR mesh geometrically closer to the input shape [FSK04]. This method optimizes the trade-off between compactness (a maximal number of *normal* vertices) and shape fidelity during remeshing. Despite this, no bitrate-PSNR curve is shown in the paper to confirm that this trade-off produces better coding performances than [GVSS00]. Also, nothing has been addressed concerning the MR analysis scheme to use (as discussed in section 3.1.1).

Another way to achieve compactness is adaptive SR remeshing. In this context, some triangles *are not* subdivided down to the finest level. From a pure geometric standpoint one can consider that the triangles *are* subdivided (using mid-point interpolation) but no details are added to the new vertices. In this way, adaptive meshes can be regarded as uniform meshes with many zero wavelet coefficients in their MR representation.

4. Components

As introduced in section 2, all SR remeshing processes share a common structure (figure 2). The first need is a tool for building the base mesh. Then, at each iteration, positions must be defined for the newly inserted vertices (geometric fitting stage). It generally leans on a parameterization that maps the base mesh onto the IR mesh.

The success of a method does depend not only on its individual components but also on their interaction, all the more so as they can not be exchanged separately. Thus discussing them independently can not tell the whole story. However it gives insights into their contribution to the achievement of the remeshing goals, and thus improves the understanding of individual methods. In this section the alternatives for each component are described and discussed. The relevance of these alternatives must be measured in terms of efficiency of the resulting SR mesh for the applications. To this end we refer to the remeshing goals defined in section 3. For an immediate overview of the choices all the SR remeshing methods made for the implementation of each component, the reader can consult table 1 (column 3), page 6.

4.1. Building the base mesh

The base mesh must be chosen wisely since the whole algorithm relies on its refinement. A suitable base mesh is one that allows for building a good SR mesh, with respect to all criteria (fidelity, quality, and compactness). Thus it depends on the following stages: parameterization, refinement and fitting. However some criteria can be exhibited [BMRJ04, PPT*11].

Topology. The base mesh must have the same genus and boundaries as the IR mesh.

Shape of the triangles. Provided that the patches can be parameterized onto the base triangles with low distortion (see discussion in section 4.2), well-shaped base triangles with homogeneous sizes contribute to well-shaped SR triangles and to uniform sampling.

Approximation. The base mesh is expected to coarsely approximate the IR mesh. The first reason is to make easier the building of the patches and the parameterization when combined with the projectability criterion (see below). Now assume that the base mesh and the LR mesh are similar, which especially holds when using an interpolating subdivision scheme. A second reason is that the LR mesh may be used as an approximation for level-of-detail applications. Third, a deeper investigation reveals that it may improve compactness. If the subdivided mesh well approximates the IR mesh, then the wavelet coefficients will be small.

Projectability. Most of the IR mesh can be reached from the base mesh by projections along the normal. This is crucial for normal meshes [GVSS00] and displaced subdivision surfaces [LMH00, PPT*11] and so it improves compactness. More widely it makes easier the definition of the patches and the parameterization. However it does not prevent distortions, especially if the corresponding patch is far from flat.

Regularity. There must be as few as possible irregular vertices, since they pose distortion and smoothness problems for the parameterization (see section 4.2). Then, they hamper compactness and triangle quality.

Triangle count. A low number of base triangles improves efficiency of some applications (*e.g.* level-of-details). One could also expect it to improve compactness because less connectivity has to be coded. Actually the exact count has little impact, as long as it is reasonable (a few hundreds triangles). The first reason is that fewer triangles imply larger triangles and patches which may contain more complex geometry, so projectability is reduced. Secondly a much more important criterion for compactness is regularity.

In order to meet these goals, three different approaches have been used:

- The base mesh results from an incremental simplification. The patches are updated during the process.
- The patches result from a partition of the mesh. The base triangles are defined accordingly.
- A global parameterization of the IR mesh is computed first. Patches and base triangles are defined simultaneously by triangulating the parameter domain.

4.1.1. Incremental simplification

The first approach defines the base mesh as the result of a greedy algorithm that incrementally simplifies the IR mesh. A local operator is applied, such as edge collapses [GVSS00, LKK03, AGL06, PTC10] or vertex removals [LSS*98,KLS03]. The IR vertices are tracked during the simplification such that their position with respect to the base mesh is known. This approach makes it easy to ensure that the base mesh and the IR mesh have the same topology: well established local criteria ensure that each local modification preserves the topology. In addition it provides control on the generation of the base mesh: the priority criterion can be chosen freely, *e.g.* such that geometry, curvatures, or features are preserved. The criterion can also take into account the shape and size of the base triangles. It may even predict the parametric distortion [KLS03,PTC10].

4.1.2. Mesh partitioning

The second approach is illustrated by figure 6. A Voronoi diagram [EDD*95] or a centroidal Voronoi diagram [Gus07, KPA10,CJL11] is computed on the IR mesh. The base mesh is then defined as the dual of this diagram. It is efficient in generating base triangles of uniform sizes and good aspect-ratios (property of Voronoi diagrams). On the contrary, enforcing the topological constraints is not trivial for surfaces of complex topology. In practice Voronoi cells may indeed not be homeomorphic to a disk: for instance they may be ring-shaped around a tubular part, or encompass a small handle. This problem is overcome in [EDD*95] by incrementally adding Voronoi seeds until all cells are homeomorphic to disks; and in [Gus07] by adding seeds near the offending cells. A convincing alternative has been proposed for mesh simplification in [BA09]. Since Voronoi seeds end up being uniformly distributed, the base mesh quite well approximates the IR mesh. It can further be improved by weighting the diagram or by carefully choosing the seeds (by placing some along the sharp edges [CJL11] for example). Once the base mesh is defined these methods may further face problems in computing the patches, for instance geodesic intersections [EDD*95].

4.1.3. Triangulation of the parameter domain

The third approach consists in defining a coarse triangulation in the parameter domain. In [Gio99] the IR mesh is first partitioned by a region growing procedure. Each region is then flattened and triangulated independently (thanks to a constrained Delaunay triangulation), leading to possibly elongated base triangles. By contrast [KVLS99] defines the patches through a spherical parameterization of the IR mesh: some initial patches are defined in the spherical domain; those patches are further split and optimized with a relaxation procedure, in order to approximately equalize their areas. Some methods build a planar global parameterization of the IR mesh: in [HLG01] a greedy heuristics tries to balance patch areas and angles; in [LYHL06] the patches are defined as the dual of a 2D centroidal Voronoi diagram. Good aspect-ratios and uniform triangles can be reached by uniformly positioning the base vertices in the parameter domain [KVLS99, HLG01]. However it is required to first build a global parameterization with low distortion, which is known as a hard task, especially for high genus meshes [SPR06].

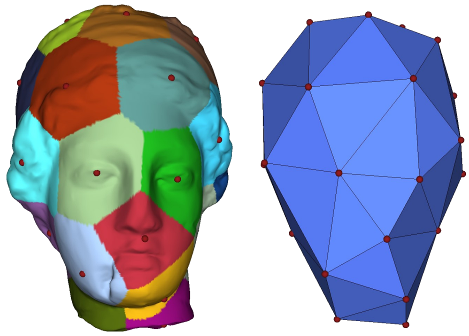

Figure 6: *Building the base mesh (right) as the dual of a Voronoi tessellation (left).*

4.1.4. Discussion

The literature related to mesh simplification is large [CMS98, Lue01, BKP*10] and it may refer to these techniques with different vocabulary. Simplification or decimation generally designate any process for building a simplified mesh. It includes methods based on incremental simplification, partitioning (of the surface), and volume clustering. The latter designate techniques where the embedding space is partitioned into cells: all the vertices in one cell are clustered and merged into a single representative vertex. However these methods are not suited to our context: their main advantage is to be fast, but they provide few control on topology and shape of the triangles. Another common classification distinguishes local and global methods. Other papers mention re-sampling or mesh approximation.

Going back over the criteria listed above, one notices that regularity (of the base vertices) and triangle count are not much considered in the literature. It is likely due to the low number of base triangles and irregular vertices (usually several tens at most). This leaves even less room for adjusting vertex valency since it is globally constrained by Euler's formula. A growth of the triangle count has several consequences: it may damage regularity and parametric smoothness, possibly impacting the quality of mesh elements; it leaves more room for adjusting valency and managing smoothness; approximation and projectability could be improved, possibly increasing shape fidelity and compactness; more connectivity has to be encoded, possibly decreasing compactness. To our knowledge it has never been closely studied. It would worth it, and could lead to effective criteria.

Even if the triangle count is low the place of the irregular vertices may be important. The valence of a base vertex indeed impacts all the incident patches. So both distortion and smoothness of the whole patches can be improved if the irregular vertices are carefully placed with respect to the geometry. An insightful discussion about angle and area distortion can be found in [PTC10].

4.2. Parameterization

The geometric fitting defines the position of the vertices for the intermediate SR meshes (see figure 2). It always relies on a parameterization in order to define the relation between the SR and IR meshes. It is a one-to-one mapping from the base mesh onto the IR mesh, matching each base triangle with the corresponding patch. It may be global or local (per patch), and explicit (precomputed and stored) or implicit (computed on request). Much attention has been paid to the parameterization because it greatly impacts on the quality and compactness goals (sections 3.2 and 3.3). It may also be useful for later texture mapping. Two criteria are usually studied for analyzing a parameterization.

Distortion. A parameterization with no distortion at all does not deform triangles. Only an isometric map achieve this. Thus a first way of measuring distortion is the preservation of lengths. Assuming that the base triangles are well shaped, an isometric map would produce well-shaped triangles and uniform sampling. A second measure is the preservation of angles: a conformal map produces well-shaped triangles. A third measure is the preservation of areas: an authalic map produces uniform sampling, but does not prevent anisotropy.

Smoothness. It refers to the continuity of the parameterization. At least the first derivative is expected to be continuous, *i.e.* the tangent fields defined by the Jacobian matrix. Global smoothness tend to produce a sampling which is locally uniform, and globally has smooth gradation. It does prevent neither anisotropy nor bad-shaped triangles.

The early methods define local parameterizations. Topological problems must first be overcome to partition the IR mesh into patches [EDD*95] or regions [Gio99] which are then parameterized independently using harmonic maps (aiming at preserving the angles). The principle is to first map each patch boundary to the three edges of its corresponding base triangle and then optimize the parametric coordinates of all interior vertices. The smoothness of the transitions across them is not controlled, leading to visually disturbing discontinuities (see discussion below).

When incremental simplification is used to build the base mesh, initial parameterizations are defined during the simplification process. It is generally based on conformal maps [LSS*98, KLS03, AGL06, KPA10] or shape-preserving parameterization [GVSS00]. Here topological problems are easily solved: well-known simplification criteria ensure a correct partition of the IR mesh into patches. Contrariwise it is difficult to avoid triangle fold-overs during iterative local projections, which is a mandatory condition to define a bijective parameterization. Since each patch is parameterized independently on a base triangle, transitions across patches are not yet smooth.

Without further optimization, artifacts may appear across the patches. They are due to discontinuities in the tangent fields. As shown in figure 7, one can make out patch boundaries from the color-coded discontinuities. This drawback is reduced in [LSS*98] by a Loop subdivision in the parameter domain. It has been further improved in [KLS03] by using smoothing transition functions across the boundaries, and a local relaxation at patch corners. The resulting parameterization has a better overall smoothness (see figure 7) and less distortions, and so it is called "globally-smooth". It tends to improve gradation of the sampling. The reason is that the sampling is uniform in the parameter domain thus the smoother the parameterization the smoother the changes in sampling density in 3D.

All those methods use *ad-hoc* rules on the patch boundaries. Moreover, none of them avoid discontinuities at extraordinary vertices. This problem is addressed in [Gus07] by using a so-called manifold structure: umbrellas around extraordinary base vertices are flattened and smoothly parameterized. It is also solved by Pietroni *et al.* by iteratively optimizing the parameterization during the simplification process [PTC10]. They use so-called abstract domains, meaning that only the topology of the base mesh is considered (the mesh has no 3D embedding). Discontinuities are avoided because optimizations are performed in turn on patch-centered, boundary-centered, and corner-centered partitions of the domain.

An interesting direction for future research is to better take into account the subdivision scheme when optimizing the parameterization. It has been shown in the context of quad subdivision surfaces [HSH10] how it can help to reduce the distortion.

In many methods the parameterization is not the core of the remeshing process. It is rather used as an initial matching between SR and IR meshes. The geometry is then optimized in order to improve the compactness of the representation [GVSS00, LKK03, FSK04] or the sampling [KPA10] (see discussions in sections 3.3 and 4.3 respectively).

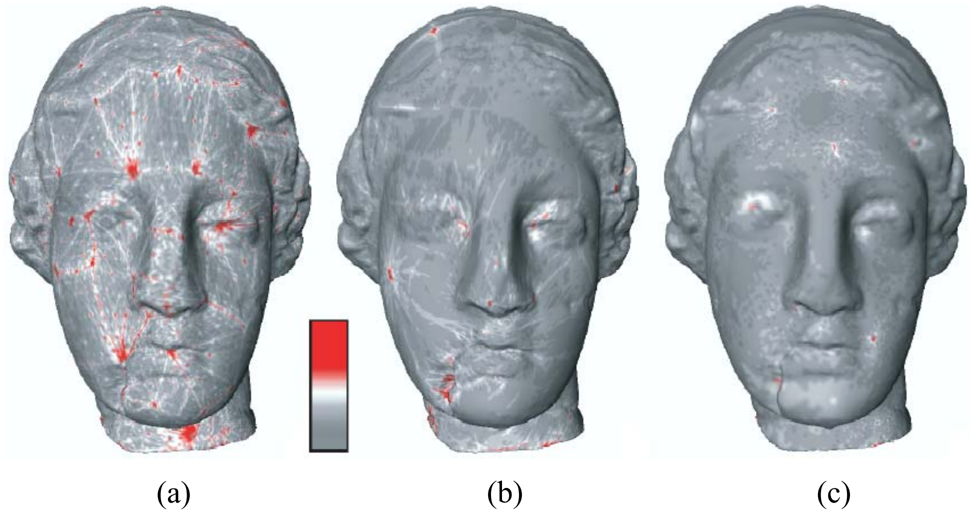

Figure 7: *Comparison of different parameterizations: (a) [LSS*98], (b) [GVSS00], and (c) [KLS03]. Colors show discontinuities in the vector field of tangents to iso-parametric curves (gray: small magnitude; red: large magnitude). Image of [KLS03].*

4.3. Geometric fitting

After each refinement step, a 3D position has to be defined for the newly inserted vertices, according to the IR geometry. They may be placed on the IR triangles, on the IR vertices, or off the IR mesh.

The issues can be made clear from a sampling point of view. The SR mesh can be considered as a re-sampling of the IR mesh. It tends towards uniformity (so-called "regular sampling" in signal processing) because of the smoothness of the parameterization combined with uniform refinement. Thus it is prone to aliasing artifacts which are well-known in digital signal processing and in rendering. Furthermore SR meshes are designed for multiresolution analysis: fine levels aim at capturing high frequencies (HF) of the IR mesh while coarse levels aim at capturing low frequencies (LF). In light of this, two types of aliasing artifacts are noteworthy:

- Fine levels fail to capture HF. A special case is sharp edge beveling, which is analogous to staircase effect in rendering (see figure 4).
- Coarse levels do capture HF in addition to LF.

Most of the methods *interpolate the IR triangles*, which ensures that the SR mesh is "close to" the IR mesh. It is attractive because the parameterization [EDD*95, LSS*98, Gio99, KVLS99, HLG01, KLS03, AGL06, Gus07, PTC10] or the projection algorithm [GVSS00, LKK03] directly defines a point on an IR triangle for any SR vertex. However such interpolation may cause aliasing artifacts of both types. The basic cause for the first type is that the sampling is uniform at a frequency similar to the maximum frequency of the IR mesh. It is reduced in [DMS10, KPA10] by *interpolating the IR vertices*, which essentially breaks the uniformity. This is done in [DMS10] by a post-processing: the SR vertices obtained by [Gus07] are displaced onto IR vertices. In [KPA10] the authors interpolate the IR vertices at each stage of the refinement process. However the second type of artifact is still visible. In [FSK04], the authors struggle against artifacts of both types by *approximating* the IR mesh at any intermediate level. As illustrated on curves figure 8, the intermediate meshes better approximate the shape, and high frequencies are better captured at fine levels. This method is efficient because a surface to surface distance is minimized, which places the vertices at averaged positions and acts as a low pass filter as long as the SR sampling is not dense enough.

In spite of it, approximation is by far not the most popular method. There are several reasons for that:

- The remeshing algorithms somehow anticipate on the wavelet scheme that will be chosen for the MR analysis. Most people prefer interpolating schemes (actually based on Butterfly subdivision) because the analysis process is stable. Approximation (during remeshing) would be much more interesting if predicting an approximating MR analysis (useful for LoD applications) but this hard task has not been addressed yet.
- Sharp features pose problems. They contain high frequencies so they should not be present in coarse levels, but captured in MR details. Hence approximating the IR mesh seems appropriate. This holds from a signal processing point of view and for compression applications where quadratic error is measured. However, when rendering LoD for instance, the user may expect the sharp features to be preserved at coarse levels because it constitutes a fidelity criterion (see section 3.1) so interpolation is more appropriate in this other setting.

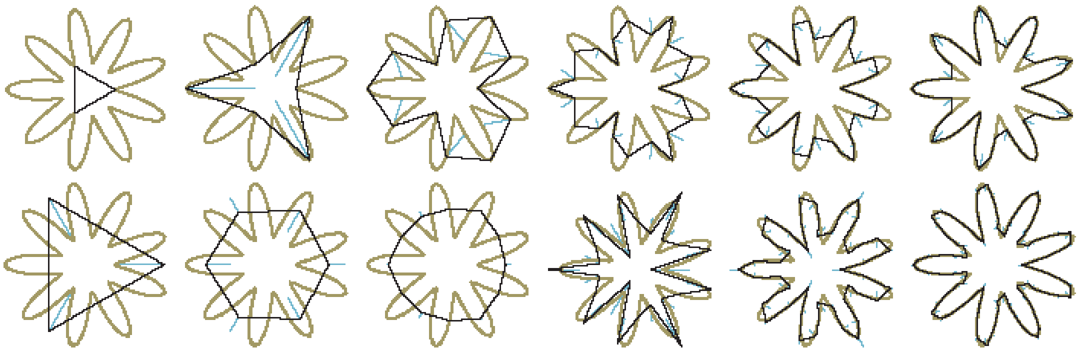

Figure 8: *Interpolating normal curves (top) versus approximating normal curves (bottom). Image of [FSK04].*

- The choice between interpolation and approximation is an ill-posed problem. As mentioned in section 2.1, a sound theoretical study in terms of sampling and space-frequency analysis would resort to wavelet bases and a limit surface, and thus would depend on the MR scheme used.

5. Input and output

When choosing a SR remeshing algorithm, any user or programmer needs to know what input/output meshes can be treated/computed. Abilities and limitations may relate to the topology, the size, or the geometry of meshes. This section is intended to be factual and descriptive since goals (section 3) and technical choices (section 4) have been previously discussed. We provide an immediate overview of the input and output properties of all the SR remeshing methods in table 1 (column 4), page 6.

5.1. Topology

The algorithms mainly differ in their ability in managing boundaries and arbitrary genus. All the algorithms presented here require the input to be manifold and produce a manifold as well. Input and output always have the same genus and boundaries. Almost all methods treat arbitrary genus meshes. Only [KVLS99] and [HLG01] require the input mesh to have 0-genus which is an important limitation. Open meshes are required in [HLG01], whereas closed ones are required in [KVLS99] and [GVSS00]. The latter has been then extended to open meshes [LKK03]. Boundaries are also handled in [EDD*95, LSS*98, Gio99, KLS03, FSK04, AGL06, Gus07].

Another topological issue is adaptive meshes. As discussed in sections 3.1.1 and 3.3, adaptive remeshing was considered in [LSS*98, GVSS00, HLG01, KPA10] as a way to reduce remeshing error and to improve compactness. However many users would prefer uniform meshes because data structures and algorithms are simpler. In that respect, $\sqrt{3}$-subdivision could have utility since it is well suited for adaptive meshes, it can support multiresolution [WQS07], and non-adaptive remeshing has already been studied [VRS03]. A broader perspective is the use of any alternative to 1-to-4 primal subdivision.

5.2. Scalability

The scalability of a given method is its ability to manage very large meshes.

Meshes containing up to several billions triangles are nowadays available. Out-of-core treatment and rendering techniques have been developed because neither main nor GPU memory is large enough to store such huge data. Since the MR structure of SR meshes is especially suitable for massive models, out-of-core processing is a key issue for future remeshers. So far it has been addressed only in [AGL06] which is an out-of core extension of [LSS*98].

Memory management is however not sufficient to evaluate the scalability: both computation times and complexity analysis are also useful. Since remeshing is usually processed off-line, the actual computation times are not the major issue: most methods try not to exceed a few minutes. A key to the scalability is the algorithm complexity but its study is often missing. Recent papers achieve faster processing times via algorithmic or numerical optimizations, such as a hierarchical search-tree structure in [DMS10] or an optimized relaxation process in [CJL11].

5.3. Geometry

A distinguishing geometric property of output meshes is the sampling. Isotropic meshes with smooth gradation are well suited for compression, simulation, and rendering (as stated in section 3.2). For complex shapes with small handles or holes, non-uniform sampling with smooth gradation avoids a too fine base mesh while preserving well-shaped triangles. On the other hand, anisotropic meshes may improve shape fidelity, but since most algorithms use smoothing and relaxation algorithms (that tend to make the output isotropic), only one method controls the anisotropy of the SR meshes [Gus07].

Concerning input meshes, more interesting is the robustness to uneven geometry: presence of noise, badly shaped elements, reversed triangles, self intersections, etc. It may originate from acquisition devices, reconstruction, or previous processings. As far as we experienced, uneven input tend to produce unpredictable output. The current solution consists in a pre-processing for cleaning the input. Making remeshers more robust may seem beneficial. One should however be careful not to decrease performances and/or ease-to-use by combining cleaning and remeshing into a single black-box. More conveniently, documentation concerning the robustness and appropriate cleaning would benefit the user. To this end, test protocols remain to be designed.

6. Summary

In this paper, we surveyed SR remeshing from the beginning in the mid-1990s. We proposed different classifications of the algorithms according to their goals, to the technical components they are made of, and to the type of input/output meshes they manage. These three standpoints provided a structure for discussing the issues, describing and comparing the methods, and highlighting some perspectives. As take-home messages, we will now review the most salient issues debated in this survey.

To have a clear overview of the scene, we will first remind the reader of a few overall ideas:

- Three main trends emerge from the history (section 2.3). They are roughly related to different issues, in chronological order: topology and compactness, parametric smoothness, and sampling.
- Three main components (see section 4: base mesh, parameterization, and geometric fitting) interact to reach remeshing goals (section 3).
- Two approaches sometimes complement each other and sometimes conflict: i) the signal processing standpoint which is rather related to sampling and to the compactness goal, and ii) the geometry processing standpoint which is rather related to patches, to the shape of elements, and to sharp features.

To then ease the reading of related papers, and to deeply understand each remeshing method, one should keep in mind some underlying ideas that are however not always explicit. Here are some keys:

- Level-of-detail applications and compression are the major strengths of SR meshes. So compactness is an important SR remeshing goal (sections 3.3 and 4.2).
- The base mesh and the LR mesh have the same connectivity but they may differ in their geometries. The same distinction has to be made for intermediate levels between the remeshing and the later MR analysis. This is addressed in sections 2.2, 3.1, 4.1, and 4.3.
- According to subdivision and wavelet theory, a limit surface underlies the high resolution mesh but it is generally ignored. This may hinder good understanding of issues related to smoothness and sharp features (sections 2.1, 3.1 and 4.3).

Finally, one may be interested in programming some methods or designing a new one. To anticipate some of the technical difficulties, we will summarize a few recurrent issues:

- The patches and their layout play a central role (sections 2.2, 3.2, 4.1 and 4.2)). They link up the base mesh and the parameterization together. A lot of effort has been expended in their design and optimization, especially at their boundaries and corners.
- Sharp features constitute a major problem, sometimes ill-posed, and often conflicting with other goals (sections 3.1 and 4.3).
- Compared to general remeshing the SR setting is especially disrupted by small topological features (such as handles and holes) because the base mesh has to catch them while having a low triangle count (section 4.1).

7. Perspectives

Some open issues have been mentioned throughout the paper. We will end by further discussing several perspectives that we believe to be worth for future research.

One area for future research is to formalize the evaluation of the methods in order to make it more systematic. We noticed that remeshing goals may be conflicting and that, even considered separately, the achievement of each goal is not uniquely measured (see for instance the remeshing error section 3.1 and the shape of the elements section 3.2). So the purpose is not to design a universal measure but rather to set up a framework for evaluating and comparing the results. As for scalability issues (see section 5.2), the methods themselves are concerned, and not only the results. In this respect we suggest two main lines. Firstly the relation between measures and actual applications could be further investigated. For instance an opportunity arises with perceptual approaches for rendering applications (see section 3.1). Secondly theoretical studies are still very weak, for instance concerning guarantees on the results.

A user-friendly remesher would treat a large variety of meshes with minimum user interaction. Despite important efforts about their stability, the remeshing processes are however still far from automated. Indeed, it generally consists of several algorithms, each of them possibly requiring fine adjustments. In particular building the base mesh is delicate and it greatly impacts the output. Moreover, we noticed that a trade-off between goals is necessary (fidelity, quality, compactness), which may depend both on the type of mesh and on the application. In this context, we believe that full automation is not only very ambitious but even not desirable. Instead, semi-automated methods could replace the tuning of non-intuitive parameters by high-level user interaction. It has been demonstrated for quad remeshing [TPSHSH13] how sketching on the surface can guide the remeshing process: one can thus benefit from expert user know-how while sparing tedious tasks.

A comparison with quad remeshing opens up new prospects. Considerable advances have been made recently in quad remeshing [BLP*13]. They are widely supported by efficient parameterization methods and geometry processing tools, such as measures of differential properties. Triangle remeshing less profited from it. One reason is that surfaces are 2D objects (usually embedded in $\Re^3$), and quadrangles have 2 main directions while triangles have 3. It can be seen when *modeling* the surfaces: quadrangles are a basis for tensor product modeling. It is well known in geometric modeling that tensor product models for parametric surfaces can be easily generalized to any dimension, and so do wavelet theory and multiresolution analysis. It can also be seen when *measuring* geometric quantities. A typical example is the curvature tensor: it exhibits two orthogonal main directions, which are good guides for the alignment of quads with right angles. Transposing such reasoning for triangles is not trivial, but an hopeful investigation was recently proposed by Nieser *et al.* [NPPZ12]. This paper presents a geometry-aware hexagonal global parameterization whose gradients are fitted to smooth six-way rotational symmetry fields. The ability to align the fields with the most appropriate principal curvature direction is operated to generate meshes that present a good trade-off between shape preservation, good triangle aspect-ratios, feature-aware triangle alignment, sizing and control of irregular vertices. All these properties are also desirable for SR meshes.

In addition to geometry, surface meshes often support other data. For instance normals, color, or texture coordinates are common attributes in computer graphics. Many attributes are compatible with SR meshes and multiresolution algorithms but so far they are not managed by remeshers. Thus they have either to be re-computed, or to be transfered afterwards from the IR to the SR mesh. The user would be relieved of this if all the attributes were managed by the remesher similarly to geometry. It could also improve the consistency between the mesh structure (connectivity) and the attributes, which can be of importance for further processings.

All the skills concerning *re*-meshing could be also used for *direct* semi-regular *meshing*. Indeed, IR meshes generally result from the meshing of other data, such as point clouds, volume data, implicit or parametric surfaces. It sometimes involves a long pipeline with tedious processes (registration, cleaning, simplification, etc.). Remeshing is one more process, which may be considered as an obstacle to the use of SR meshes. By shortening the classical pipeline, direct SR meshing would not only promote the usage of SR meshes, but also avoid error accumulation and support the additional attributes more efficiently. In return, such approaches may have to cope with problems that are usually treated by previous processing steps, including noisy data, missing data, and inference of topology. Until now, direct SR meshing has been scarcely studied. To our knowledge, meshing has been proposed from volume data by coarse-to-fine extraction of iso-surfaces [WSBD00, HLMG02], and from point clouds [JK02, BHGS06]. Future works in direct SR meshing could be also inspired by works such as [PTSZ11], which presents a global parametrization from a set of range images that allows to generate SR quad meshes. This approach is interesting not only because 3D acquisitions devices provide range images, but also because they are easy to obtain from other types of geometric data (e.g. implicit surfaces, CSG, B-rep, etc.) by using standard rendering techniques.

8. Acknowledgements

We gratefully thank the anonymous reviewers for their precious feedback.

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